Professor Arie Dubi
Grand Fellow of the MIRCE Akademy

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Analytical Principles & Monte Carlo Methods

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Content

Chapter 1: Truth and Science

1.0 The whole truth

Many years ago, at the break of Tuesday dawn, I found myself lying in a hole in the ground, along with three other fellows, in the outskirts of Kibbutz Dan in the north. The early sunrays had not yet reached us and from the direction of “Tel Hamra”, then in the Golan Heights of Syria, a barrage of 120mm Mortar shells was heading our way. The three fellows were paramilitary academic cadets and had not yet worn uniform. I was a first year physics student and a fresh reserve soldier – a young, frightened paratrooper. Random events of war brought the four of us together and at random we could have all met our end should a single shell have landed in our hole. There is something particularly awful about a mortar shell, the whistle – it is a low pitched windy sound that increases in volume and pitch until it tears ones ears and freezes the blood. It is the sound of death searching for your wretched soul.

This went on continuously for fourteen hours, from the early hours of the second day of the “Six Days” war, till the evening. Slowly but surely horror started taking its toll. One guy had his jaw trembling so fast and strong that it threatened to break it. This is, as I learned later, a well-known phenomenon of extreme prolonged fear – the others breathed heavily and one complained of pains in the chest, again a known phenomenon that can lead to heart failure. As things were, I became, without my will or choice, the leader of this poor gang laying hours in a one-meter diameter hole in the ground (and, boy, was I happy it was not any bigger). At a certain point I had realized that something must be done to reduce the level of anxiety and avoid serious consequences. I tied the jaw with a sleeve of my shirt, and then, quietly and most seriously, I explained that the mortar shells are flying faster than the speed of sound, hence, when we hear the whistle the shell has already passed us. I recruited all the possible authority in my voice including the fact that I am a potential physicist, to tell them a story that was a complete lie – from beginning to end. Mortar shells fly much slower than the speed of sound. Following that story the state of the group improved miraculously. Every time the terrible whistle was heard (about every ten seconds) a sigh of relief was heard. I, of course, suffered twice, not only did I tremble with fear just as before but now I also had to fake sighs of relief so as not to betray the lie.

I believe they know the truth today. I hope they do not hold it against me – I meant well. I also thought that if I am proven wrong, that is, if the end of a whistle is an explosion in the hole, well, they will not be able to complain against me, at least not in this world.

Why do I start a discussion about "truth" with a story about an outright lie?

There are two reasons. First, this story demonstrates that even the clearest lie will be favorably accepted when the conditions are right for it. We tend to accept lies if they fit what we wish to hear.

Second, and more important is that I believe that the "Truth" is probably the single most important concept in the web of our morality. Yet, I am afraid of dogmas and fundamentalism. Sometimes one should lie, sometimes one must lie – but, only under extreme conditions and with a clear intent to do good for others possibly at ones own expense.

Having been so clear about a lie it is reasonable to ask: What then is "truth"?

The dictionary writes about "truth" the following: "Something correct and loyal, something that can be believed. The opposite of lie"

That does not help much in understanding the definition of "truth". It verbalizes a general understanding we all have as to what truth is. But it is in no way a definition. In fact, any attempt to define "truth" is bound to encounter extreme difficulties. One very soon realizes that there cannot be a single definition of "truth". In fact one can identify at least four categories of truth.

1.1 Mathematical Truth

The simplest type of truth is "Mathematical truth". Its uniqueness lies in the way it is constructed. First a number of statements are set. Such statements have two properties:

They are assumed to be pure truth and do not require proof.

They cannot be proven and no one will ever be able to prove them.

Such statements are called "Axioms". Once the basic set of axioms is laid "truth" is defined as:

Any statement is true if and only if it can be proven from the basic axioms i.e. if it is a logical outcome of the axioms.

Here is a partial list of basic mathematical axioms (1) as outlined by the German mathematician Hilbert (2).

There is one and only one **line** passing through any two given (distinct) **points**.

Every line **contains** at least two distinct points, and given any line there is at least one point not on it.

If A,B,C are distinct points on the same line, one of the three points lies between the other two.

(Attributed to the Greek mathematician Euclid) – Given a line L and a point A not on the line, then one and only one line can be passed through the point A that will not intersect the line L, in fact, the well known statement that “Two parallel lines never meet”.

Altogether there are 15 such axioms.

None of the above statements <u>can be</u> or <u>need be</u> proven. One may note that within the axioms concepts such as “Point”, “Line” and others (bolded) are used. These concepts are again not proven or defined. Their existence is axiomatic and one may state that they are defined by their use in the axioms. That is, any two concepts that have the properties described by the 15 axioms can be viewed as a point and a line respectively. Axiom 15 (not quoted here) guarantees the existence and nature of the “Continuum of points” which is an essential feature of the real numbers.

The type of truth defined above has a number of interesting characteristics. First, it does not apply to all possible statement. There are many statements that do not result from the axioms and do not negate them. The statement “Jennet is a beautiful girl” or the statement “The moon inspires young lovers” does not result from any axiom nor does it (so we hope) contradict any axiom. Thus, “mathematical truth” creates a natural trichotomy. It divides all the statements into three exclusive realms or groups:

- “True” statements, namely, those that can be proven from the axioms
- “False” statements, those that contradict any of the axioms
- “Not related” – those that do not result from, and do not contradict, any axiom.

The definition of “mathematical truth” a priori limits the realm of application of mathematics.

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1.2 Physical truth

A second category is “Physical truth” or “Scientific truth”. This “truth” is fundamentally different from mathematical truth. Indeed there are axioms in the physical theory. Yet, the definition of truth has nothing to do with them. Physics, unlike mathematics has a lot to do with the universe in which we exist. In fact, physics is about this universe and its rules. The definition of physical truth is both simple and deep. It centers on the “physical experiment” and states:

A statement is true if and only if it can be verified in an objective scientific experiment i.e. if the results obtained from the statement fit those of the experiment.

The nature of physical axioms is different from that of mathematical axioms. These axioms are not generated from inward thinking that can be done in a closed capsule. Rather, they are a result of observing the universe around us and making assumptions about it. We will next consider some of these axioms and deduce some of the, well known, physical laws resulting from them.

<u>Homogeneity of time</u>: Consider a physical experiment such as the free fall of an object of mass m, done at any point is time. One can watch the experiment, one can measure the velocity of the falling object at various points and one can see the object hit the floor. It is quite obvious that if exactly the same experiment will be performed one hundred years later the observer will see and measure exactly the same results. This is quite trivial – or is it? How do we know that the observer will see and measure exactly the same results? How do we know that the laws of physics do not change over time?

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1.3 Religious Truth

The third category of "truth" is the "religious truth". This truth in general is similar to the mathematical truth in that it has an axiom. The existence of an omnipotent divine entity is axiomatic in every religion. Yet, this truth differs from the mathematical and physical truths in two critical aspects. First, there is no trichotomy. Nothing is beyond the realm of religious truth. Every aspect of life and of the universe is contained in the religious truth. The second critical element is that unlike mathematics or physics, religious truth does claim about itself "I am the truth". This manuscript is not about religion. The religious truth is mentioned only for completeness

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1.4 Definition of Science

At this point one can define what science is. A possible definition of science could be:

Any realm of human activity in which two conditions are fulfilled:

A systematic study of laws and rules that enables prediction of future behavior of systems to be done

A definition of "truth" exists which is independent of human beings.

In the spirit of the uncertainty described above I would not mind to have other opinions on the definition of science. Furthermore, according to this definition literature will never be a science. Politics is not a science, unless one adopts a definition of truth as “the truth is what I say”, which immediately negates the above definition.

However, being a science does not make anything good or bad. Nor does it put any discipline higher or lower on any ladder of values. It is just a matter of definition.

Indeed the above discussion does not solve the question of the fourth truth, that of our life reality, of life and everything in it. It does, hopefully, add just a bit to the clarification of the concept of science and the complexity of reality.

It is at this point that one can ask the question: *Is system engineering a science?*

Chapter 2: About the process

2. 0 Transport of particles

The road we are taking should lead us into system engineering. The path started with a definition of the truth and our next step involves a look at neutron behaviour. The reader will, hopefully, be happy to note that he needs to know very little about neutrons in order to understand the principles of their behaviour and it is those principles that are of interest to us in this discussion.

A neutron is a very small particle. It moves in straight lines since it is neutral and it collides with the nuclei of the medium in which it moves. Upon collision a number of “nuclear reactions” can take place. Among these reactions there are: elastic scattering – in which the neutron bounces off the nuclei. The kinetic energy is conserved and shared between the neutrons kinetic energy and the nuclei, much like the experiment in Figure 2. No energy is lost to the internal structure of the nuclei. Inelastic scattering is similar except that a certain portion of energy is absorbed by the nuclei and excites its internal structure. Absorption reaction is such that the neutron disappears (being absorbed into the nuclei) and in general some of the absorbed energy is emitted in the form of a photon or other particles. In general, a neutron is completely defined by indicating its location $\vec{r}$, the direction of motion $\vec{\Omega}$ and its energy (velocity) E. The neutron thus has 6 degrees of freedom (3 for its location, 2 for the direction and 1 for the energy). The vector: $\vec{P} = (\vec{r}, E, \vec{\Omega})$ indicates a point in a phase space (which is a fancy name for the three elements of the vector) in which a neutron enters a collision with a nucleus. As the neutron moves in the medium, the distance to its next collision is a random variable. That random variable has a true exponential distribution, nice and simple. The mean distance between consecutive collisions is denoted by $\frac{1}{\Sigma_t}$ and Σ_t is called “the total cross section”. Thus, if x is the distance to the next collision its probability distribution function (pdf) would be:

2.1 $$f(x) = \Sigma_t e^{-\Sigma_t x}$$

This means that the probability that the next collision will occur at a given distance x_c from the starting point and within a small interval Δx_c is given by:

2.1a $$f(x_c)\Delta x_c = \Sigma_t e^{-\Sigma_t x_c}\Delta x_c .$$

How then does nature choose the point at which the neutron will have the collision? The fact that the process is controlled by equation 2.1 and 2.1a means that nature has some way of guaranteeing that the probabilities of 2.1a will be realised. How does it do that? Let us assume that $\frac{1}{\Sigma_t} = 20_{cm}$, $\Delta x_c = 0.02cm$ and $x_c = 10_{cm}$. Equation 2.1a then states that the probability of the neutron entering a new collision somewhere between 9.99cm and 10.01cm is 0.0121. Thus out of 10000 distances between collisions about 121 should be in that range. Indeed, in N experiments the number should get closer and closer to 0.0121xN, as N increases to infinity. This is a result of statistical laws such as the Chevicheves inequality and the weak law of large numbers (5). How would nature guarantee that in 10000 experiments, about 121 will fall in that range? Does nature remember the distances in former collisions? That is highly improbable, as no such mechanism has ever been observed. Nature does not remember such past events. Rather, nature makes sure that each event is conducted such that equation 2.1a is maintained. We do not know exactly how nature does it but here is a possible way. Let ξ be a random variable uniformly distributed in the interval [0,1]. Being uniform means that its pdf is constant over that range and hence $f(\xi) \equiv 1$ and $f(\xi)\Delta\xi = \Delta\xi$ hence the probability of this random variable to take a value in any given interval is simply the size of the interval. Assuming that nature can do that one may now consider the relation

2.2 $$\xi = F(x_c) = 1 - e^{-\Sigma_t x_c}$$

F(x) is the cumulative distribution function (Cdf or cdf), of x. The value of x_c may be obtained from this equation once ξ is known in the form

2.2a $$x_c = -\frac{Ln(1-\xi)}{\Sigma_t}$$

This guarantees that the values of x so obtained have the correct distribution because equation 2.2 implies that $\Delta\xi = \Sigma_t e^{-\Sigma_t x_c}\Delta x_c$ (just take the differential of 2.2). Since the probability of x_c to be in an interval Δx_c is identical to $\Delta\xi$ it is obvious that equation 2.1a is fulfilled. This procedure is easily generalised. Obtaining x from the equation:

2.2b $$\xi = F(x)$$

Equation 2.2b implies that, for any Cdf, F(x), the probability of x to be in the interval Δx is $\Delta\xi$ which can be written as:

2.2c $$\Delta\xi = \frac{dF(x)}{dx}\Delta x = f(x)\Delta x$$

Thus, condition 2.1a is fulfilled and the solution of 2.2b for x is a proper sampling method for any distribution.

This is one option of nature to "sample" a random variable from **any given distribution f(x)**. While it is not sure that nature chooses this method to sample random variables in any natural phenomena it is clear that we can perform such sampling on a computer.

First choose a uniform random variable (using a routine that exists on every computer including some hand calculators) and then solve equation 2.2b for x_c. The random variable is then guaranteed to follow the required distribution. This is demonstrated in Figure 2.1 below. The Exponential distribution is shown (solid grey line) and a histogram of 20000 sampled points using the above algorithm is plotted (+ markers). It is seen that the histogram follows the required distribution. Indeed, the match between the histogram and the function will improve as the sample size is increased. This is guaranteed by the weak law of large numbers.

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Chapter 3: A few words more about the transport Equation

3.0 Transport and free will

It was stated in the last chapter that the transport equation (2.21) is the foundation of the science of system engineering. The general structure of the equation and its existence were explained and proven.

3.1 $$\psi(\vec{P}) = \psi(\vec{B},t) = S(\vec{B},t) + \sum_{\vec{B}'}\int_0^t K(\vec{B}',t' \to \vec{B},t)\psi(\vec{B}',t')dt'$$

Being the foundation of this science means two things. First, every system obeys this equation. Recall that this is not a single equation, this can be a simultaneous set of billions of interrelated equations. All the information about the properties of the system and its components is stored in the transport kernel $K(\vec{B}',t' \to \vec{B},t)$ and this information is sufficient to know everything about the future of the system, or any aspect of it. In fact, this means that the function $\psi(\vec{P})$ contains all the information about the future of the system, availability, reliability, spare parts required, throughput and anything else.

Whenever a statement such as "the future of the system is known" is presented, one should immediately get worried about the fate of "free will". After all, a system may incorporate human beings operating it and if we state that the future of the system is known, then we may fall into the trap of relieving the operators from any responsibility for this future. Scientifically they may claim that they were merely “Operated” by the rules of science and everything they did was predicted a priori having nothing to do with their will or intentions. This is, in fact, quite an old problem. When Newton came out with his three laws of mechanics, it was argued that if all the universe can be assumed to be made of small atoms, and if all the forces acting on these particles are known at T=0 (the beginning of the Universe) then the behaviour of the Universe, and all it contains, can be predicted by solving Newton’s equations. This notion, which appears everywhere in classical physics, actually throws the concept of “free will” straight out of the main door of philosophy. Indeed, even if these equations will never be solved by humans, they still have a solution, a unique solution, because they are second order differential equations with boundary conditions at T=0. Indeed, one may claim that we will never know the future because we will never be able to solve the equation. But, this is not about “knowing the future”! This is about the predetermination of that future. The very fact that there is a unique solution is the critical element, because if there is a unique solution it means that every event, every occurrence has been determined uniquely at T=0 and nothing we do can affect these events. In fact, everything we do is part of that unique solution. We can do only one thing and the thing we do has already been determined by the conditions at T=0. This is not a trivial point. One can easily agree that if there is no life in the Universe then the future of the Universe is predetermined by the initial conditions. Whether it expands or shrinks, creates gas clouds, galaxies, nebulas, stars and planets, everything on the time line is controlled by the initial conditions and by the laws of physics, determined also at T=0, remember the homogeneity of time. If life and intelligence are considered as just additional ordered molecules with complex interactions, just very complex machines, then they are part of this Universe just like the interstellar molecules and the future is again predetermined. That eliminates “free will”, responsibility and the ability to affect the future. This is a serious moral dilemma. It was eased by the introduction of quantum physics. Quantum physics introduced two new elements “true stochastic events” and “the effect of the observation”. The first element suggests that some of the basic physical laws are of a true statistical nature.

Consider the throw of a dice. We use this example to describe a statistical experiment in textbooks because every time the dice is thrown a different result is expected and it is a beautifully simple experiment to demonstrate the meaning of a probability function and its applications. But, is the throw of a dice a “true” statistical experiment? Not really. The reason one obtains a different result is that each throw is slightly different, the angle of throw, the position of the dice, the velocity etc. If one could repeat each experiment exactly the same then in each experiment exactly the same result would have been obtained. This creates no problem at all for the application of statistical methods. But, it raises the question: Are there true statistical experiments? Or is it the case that statistical experiments are actually deterministic experiments with many unknown and varying “internal variables”? Quantum physics sets this issue stating that “all events in the subatomic domain are of true statistical nature!”. The question of whether a neutron will be absorbed or scattered is truly a probabilistic event. The time of decay of an excited nuclei or atom is a true statistical event. This indeed means that there cannot be a "unique deterministic" future. There can be many futures and the conditions at T=0 do not define the future uniquely. Furthermore, quantum physics suggests that the state of a physical system depends on the question of whether it is observed (measured) or not. The observation of a system affects its quantum state and changes the future. Hence, since only humans perform conscious predetermined observations it is within their free will to affect the future. In the case of equation 3.1 the situation is a bit simpler but bears a number of similarities. Recall that this equation is related to the "average" event rate. In fact, it will provide us with knowledge of the average behaviour of a system. Still, if many systems are observed each one may have a "different life history". Hence for every property of the system there must be a distribution of this property. We will further discuss this when considering the concept of "warranty" but for the time being we just stress the fact that no system will have a fixed unique predetermined future. Statistical laws, expressed in the transport kernel, control the system and its life is a stochastic process. Furthermore, in creating models of systems we may introduce the human operator as part of the system. But, in our models we will leave little room for "free will". We assume that the operator will behave as instructed and as expected in good working practice. We may introduce models of human errors or even account for malfunctions resulting from fatigue or negligence, but we most probably will not introduce premeditated sabotage that is an expression of free will. If one would insist on incorporating all the variety and diversity of human behaviour into the model, then it will be easy. We will just simply state that we do not have a green clue as to how to do it. In that context it is worth mentioning the question often rose of whether equation 3.1 can be used for predictions in economics. The current answer is: no, it can not! This is for a number of reasons. In principle an economic system does obey equation 3.1 in that it can be described as an aggregate of components: human beings, machines, factories, shops and products, interacting with each other and evolving in time and in the space of the possible states of such a system. Yet, we do not have a clue as to the structure of this space, except for simple academic, non-realistic cases. Secondly, the nature of the interactions is vague. It does involve the element of human behaviour and free will (suddenly everyone buys this gadget, and then no one buys something else). At this time we are simply unable to construct the transport kernel and therefore we are not able to address the problem through this equation. I personally like it this way and I do believe that this will remain forever the case because otherwise it would mean that free will does not exist. It would mean that we have managed to "correctly" describe human beings as predictable machines. I think we will not be able to do it. I hope we will not. I hope that Asimov's science of "Psychohistory" will remain forever science fiction.

With the above reservations it is claimed that equation 3.1 does provide predictions of every aspect of the system behaviour. Let us first review how the event rate is related to practical quantities.

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4. The analysis and optimisation of spare parts and other resources

"A software package is better if it requires less data", ***Anonymous expert***

4.1 General discussion

The problem of spare parts and resources is one of the most basic questions desalt with by the human kind since the beginning of time. It is as old as human history and it is not going away. One can easily envision our early ancestors roaming the savannahs of today's Ethiopia running after the deer, and away from the lions. How long should the chase continue? How many deer is enough? Every additional hour in the open space of the flatland increases the risk of becoming food. Yet, if more dears are hunted their meat can be dried up on the cave fire and increase the chance of surviving the soon coming dry season when food is scarce? Did they ever do calculations?

There are n people in the clan. Each consumes a minimum amount of λ pounds of meat every day. The dry period extends over, say, T days, the weight of an average deer is w. Hence, We should have a total of: $\frac{n\lambda T}{w}$ deer caught. One does not think they actually did such calculations. But, they could! In fact, one can envision a gathering of the hunters at night, with the fire burning for light and protection. The naked wise guy of the clan quietly speaking about: Introduction of the variability of the length of the dry season and establishing confidence intervals for survival. i.e. treating T as a random variable. Introducing the variability of the weight of a dear into the equation. Introducing the variability of human daily meat consumption (We can make these old useless scientists eat less – Shouts the crowd). Recycling? - The possibility to go out during the dry season, and get some more food. Before we know it our ancestors would have, apart from the urgent need to obtain food, an urgent need to solve equations that we do not know how to handle even today. Indeed, eating the scientists would have both eliminated that problem and eased the food problem.

What then is the advance we made in the last three million years?

Consider the following simple example: A company operates 25 heavy machines. Each machine contains 8 identical components. If any of these components fails the machine is down. Upon failure of a component there is a probability of 0.6 that it can be repaired without replacing it (this takes 1 hour), otherwise the component must be replaced. The above components could be heavy tires on trucks. Also, a replaced tire is discarded and cannot be refurbished or used again. The failure rate of a tire is $1.2\text{x}10^{-3}$ 1/hour. How many spare tires should be stored for a period of 1 year?

Unlike our ancestors, we do not need to run after dears, or away from lions. We just go to the closest grocery and fill our refrigerator to our hearts desire. This leaves a lot of spare time that we can use to invent methods to calculate how many spare parts are needed.

We will bring a number of approaches to this problem.

4.1.1 Ancestors' approach: This method is exactly the same as the one used above to calculate how many deer are needed. n=25 trucks, m=8 tires per truck, T=8760 hours of operation. On the average, trucks operate only 16 hours every 24 hours, namely, 0.6 of the time and, thus, the average number of failed tires would be: S=0.4x0.6xλnmT or 504.0 tires. In this calculation only 0.4 of the failure rate is used as 60% of the failures can be repaired on location without a spare.

4.1.2 Modified ancestors-approach - Sufficiency: 504 spares are quite a lot of spares for a small fleet of trucks! But, we know that this number is just the average number of failures. The actual number of failure is a random variable having a Poisson distribution (Not exactly). In fact the probability that this amount will suffice for the whole period is near 50%. That means that there is a probability that we will run out of necessary spares long before the year-ends. How can we protect ourselves from this? It is easy; the probability to have exactly k failures is given by:

$$P(k\,|\,T) = \frac{u^k e^{-u}}{k!} \quad \text{where } u = 0.4x0.6x\lambda nmT$$

The probability that S spare tires will suffice for the whole year is the same as the probability that there will be less (or equal) than S failures hence:

$$4.1 \quad P_s(S) = \sum_{k=0}^{S} \frac{(0.4x0.6x\lambda nmT)^k e^{-0.24\lambda nmT}}{k!}$$

The table shows the number of spares, S and the "sufficiency, P_s (S), assigned with each quantity.

S	400	450	500	520	550	560	570
P_S(S)	$7.9x10^{-7}$	$7.76\ x10^{-3}$	0.441	0.769	0.979	0.993	0.998

Table 4.1: Sufficiency table for replacement with total condemnation of failed units.

It is seen that if we want a confidence of 76.9% that {the amount of spares will be sufficient for the whole period} we need to purchase 520 spare tires. To obtain a sufficiency of 99% we need about 560 spares.

The reader may put himself at the position of the decision-maker. How many spares would you then decide to purchase? 400, those gives almost zero confidence, or 560?

It seems to be quite logical for the decision-maker to ask (although it is rarely asked): What would be the performance of my fleet with any given number of spares? What would be the availability of trucks as function of time throughout the year? This is not difficult either:

If no spares are purchased then the survival of a single truck is given by:

4.2 $R(t) = e^{-0.4x0.6\lambda mt}$ [The probability not to have a single failure in a track up to time t]

This will be the reliability of a single truck. Since, with no spares, trucks are un-repairable, the availability and reliability are identical. Thus, this is also the fraction of operational trucks at time t.

If S spares are purchased then, assuming negligible repair time, the availability of a single truck will be unity until all S spares are exhausted and then it will decrease at the same rate as in 4.2. Hence, the unavailability at any point of time, t, is the probability density to have S failures, in the whole fleet, up to an intermediate time t'<t multiplied by the probability to have a single failure in the system in the interval (t-t'). All this averaged over t'. The probability density of having S failures in the field up to time t' is given by the Gamma distribution,

4.3 $G_S(t) = \dfrac{\lambda^*(\lambda^* t')^{S-1} e^{-\lambda^* t}}{(S-1)!}$ where $\lambda^* = 0.4x0.6mn\lambda$ and the probability of a single failure in the interval (t-t') is given by $\left(1 - e^{-\lambda x 0.4x0.6x8x(t-t')}\right)$ hence the unavailability:

$$U(t) = \int_0^t \frac{\lambda^*(\lambda^* t')^{S-1} e^{-\lambda^* t}}{(S-1)!} x \left(1 - e^{-0.4x0.6x8\lambda(t-t')}\right) dt' \quad (4.4)$$

This integral can be worked out and the results for the availability obtained are shown in Figure 4.1:

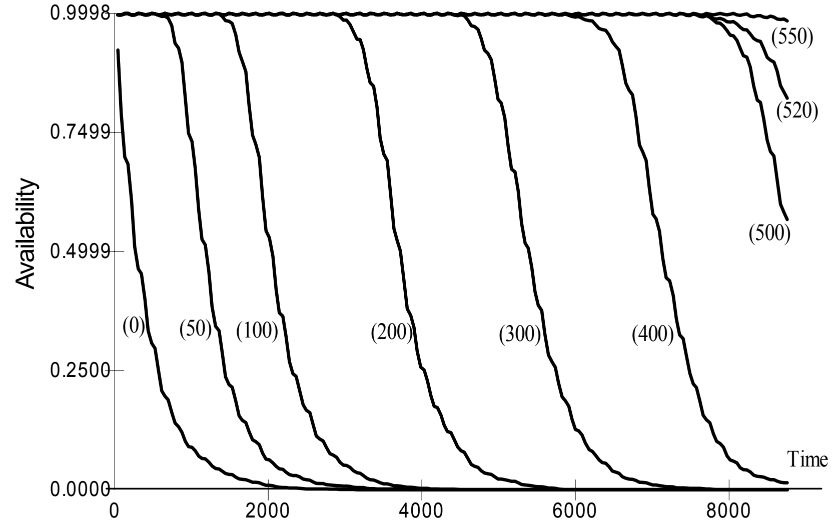

Figure 4.1: Trucks availability as function of time for various spare quantities.

For the decision-maker, the above Figure should be very useful. At this point one must decide what is the performance target of the system? How much truck does one really need in order to fulfil the goals and tasks of the organisation? After all, not all the trucks are working all the time? Let us assume that the requirement is: "The average number of operational truck should not go below 20 at any time during the year." The operators of the system must set this goal. Without it we really do not know what the system is needed for and one has no basis for a decision concerning spare parts or any other resource.

With the "system performance target" defined one can see that with 500 spares the availability drops, at the end of the year, to 0.56, with 520 spares the drop is to 0.82. Thus, purchasing 520 spares will provide the required performance.

A few conclusions can be drawn at this point:

The system does not necessarily behave the way we would like it to. The availability curve does not spread evenly throughout the year, and there is no steady state availability. The yearly average availability of the system with 400 spares is 0.82. This means that if the target would be an average yearly availability of at least 20 trucks (i.e. the time dependent availability averaged over a year) then 400 spares would be enough. This, of course, would create a catastrophe because for the last two months of the year there will be no trucks at all (see Figure 4.1). Setting systems target based on a time average could be a serious mistake. One should always observe the time behaviour of the performance measure, unless one is confident that the performance is independent of time.

It is quite tempting, and often done, to look at the sufficiency and chose a healthy confidence, such as 0.995. This would be the case particularly in organisations in which the spare parts department is responsible only for availability of spare parts and is not directly responsible for the actual operation of the system. This, in fact, includes all organisations. The spare parts department is aware of the simple fact that if spare parts are missing when required, they will look bad. If spare parts are available when required they look good. If lots of spare parts will remain in stock with no use, nobody will notice. By the time the organisation starts feeling the pain of losing a lot of money in unknown directions, and taking into account the fast rate of positions changing in today's world, there is a very good chance that the decision makers responsible for the "dead storage" will be already consulting in another company. Thus the modified ancestors-approach does have a reasonable level of success. A decision to buy 570 tires would seem quite reasonable. It is actually 50 spares more than necessary. It is an additional 10% on the budget that is actually thrown away.

Spending 10% on tires may not be a lot, but there are other spares like a jet engines at 2 million dollars each, pumps at $ 48000 each and so forth. The discussion above applies to all of them. A saving of 10% over the expenses of any industry makes a huge difference.

Can one still save more on these spare parts?

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Chapter 5: Bedtime stories for System Engineers

This chapter is devoted to stories. Stories that have been lovingly collected through some 20 years of interaction with the industry in an attempt to transfer to them some of the modeling methods described in this monograph. One may justifiably wonder why would one bring stories into a book so nicely loaded with mathematics. Yet, there is a good reason for it. The mathematics is not intended to "stand alone". It is not just for the beauty of it that this book is written, although this beauty is the one and only true personal motive in it. It is for the positive impact that the use of such modeling may have on the industry and on our lives. Imagine a possibility to reduce the cost of all industrial expenses by 2%, or by 10% - wouldn't that give us enough funding to both educate and feed properly all the children in the world and an ample amount will remain for improving the environment? Well, it is our experience that the money that can be saved is definitely in this range, may be higher. Also the examples brought herein are attempting to connect the mathematical formulation to the real daily problems of the industry.

A fighter jet cannot take a turn of 25G. – Why?

There is a human pilot, inside the aircraft, which cannot take this pressure!

It is the wonder of the human brain and body that created the industrial marvel and that limits it!

The human being is a key element in any industrial system. It is he who creates it and operates it. It is he who attempts to solve the problems of making it good and efficient and it is he who limits its efficiency and design. Admittedly, one finds it extremely difficult to bring the human into the model. The human operator can be easily modeled if his operations are predefined and he serves as an additional component with a set of predictable, possible actions. But, it is next to impossible to consider his personality, free will, likings and disliking, sympathy etc. All these indeed come into play when one attempts to introduce a new way of thinking into an industry. Being such an essential part of the industrial process the human factor deserves a very serious discussion. The author is far from being able to make any such serious discussion. So, a set of stories with just a pinch of moral in them present the best substitute.

5.1 Zero failures

I was once invited to a general meeting of the Maintenance and logistic staff of a big company dealing with Medical equipment (CAT scans and the likes of it). When I say that I was invited, I do not really mean that anybody bothered to either call me or write to me and actually invite me. This never really happened. What I truly mean is that through my ongoing connections I heard about this meeting and managed to, kind of, invite myself to give a talk.

The meeting went very well with a lot of enthusiasm and viability. The head of the relevant department started with a rock curving speech about dedication to the customers being the key to success. Above his head proudly hung a big plastic sign on which it was written “The target – Zero failures” and indeed he went long and wide explaining how important this target is.

When my turn came I looked again on the big plastic board and the mathematical genes in me rebelled. I started with the basic equation of optimization of total cost. Here it is:

$$C_T = N_f C_f + N_m C_m + T_d C_d$$

N_f is the number of failures and C_f is the cost (to the company) of repairing a failure . N_m is the number of maintenance operations and C_m is the cost of maintenance and T_d is the down time of the system with a cost C_d per unit down time. Indeed if there are various types of failures and maintenance operations then a summation sign is added and we sum over the various types of operations and failures. This expression can be further elaborated and written in the form:

$$C_T = C_f x \int_0^T \psi_0(t)dt + N_m C_m + \int_0^T C_d(t) x [1 - A(t)]dt$$

The first term is simply the integral over the event rate going into a failed state providing the total number of failures while the last integral is over the unavailability multiplied by the cost (that may be dependent on the time when the down time took place). The purpose of the organization is to reduce the total cost. In the context of the above equation we can only change the number of maintenance operations N_m. The point is that such changes will effect all three terms. By applying more maintenance the age of our components will be reduced, this will result in fewer failures and less down time. Thus increasing the cost of the second term will help to decrease the cost of the first and third term. Indeed it is possible, this way, to decrease the first term to zero – "Zero failures". All one needs to do is continuously perform maintenance. Imagine your car going into the garage for oil replacement. Once the operation is completed the oil is replaced again and again and again in a never-ending cycle. I looked at the audiences, at this point, and realized that many had a quart of laughter mixed with a pint of anger in the corners of their eyes. This kind of oil replacement is definitely extremely stupid. But, an equation is an equation. Increasing N_m such that it equals $\frac{T}{T_{mt}}$ where T_{mt} is the duration of maintenance will most definitely reduce the number of failures in your car to zero. Indeed, you will not have a car (and the third term of the equation will be very high). Still the target of:

"The target – Zero failures", will be definitely achieved.

This will cost a lot of money (in maintenance). This will make the customers extremely unhappy as they will not have the system but it will most definitely achieve the goal of the organization as, so proudly, expressed by the plastic scroll. No! it is not self evident that the system is needed. Not more then the need to reduce the number of failures. The goal of the organization must be expressed in coherent correct terms or it may lead to solutions such as the above. In fact, I said, one can even devise a way to reduce the cost of maintenance to zero without loosing the glorious target of "Zero failures". Do not manufacture or sell the system! This will indeed reduce to zero both the cost of maintenance and the cost of failures. Both solutions are points on the space of the above equation.

At this point the department head rose. I knew it was coming. I knew it. He looked at me with a polite chill in his eyes and gently stated " It is clear to everyone in this meeting that the systems must work! It is not necessary nor useful for you to suggest stupid solutions". Yes it is, I insisted. Yes, it is because the stupid solution is actually written all over the above statement. If you write "Zero failures with 98% average availability" then you define a goal that may not be achieved but makes both industrial and mathematical sense. Anything else does not belong, not to industry at least. I then continued my talk but the highlight of it was over.

I could never check if what I said made any impact because I was never again "invited" to any meeting in that plant. I do not know if they ever achieved zero failures. I do know that they achieved "zero aggravation from me". Zero aggravation seems to be a target of every well doing organization. It is a primary target of human beings.

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