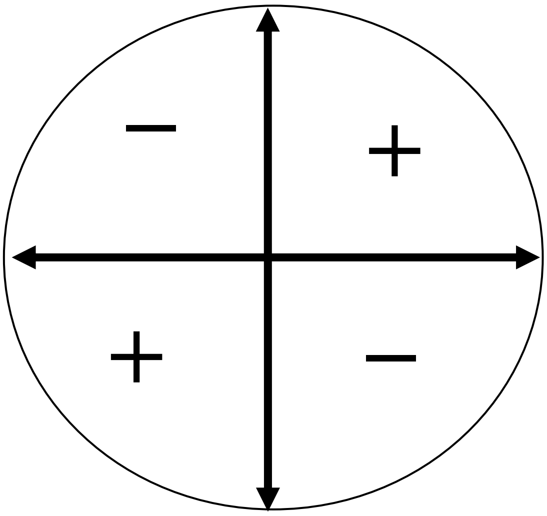
Dr Jezdimir Knezevic



Mechanics of Mathematics

Published by MIRCE Science Limited

Woodbury Park, Exeter, EX5 1JJ, UK.
Phone: +44 (0) 1395 233 856

ISBN 978-1-904848-04-2
EAN 9781904848042

To the memory of my nana Katarina,

who from the very early days of my life taught me, among million other things, the difference between adding numbers and adding physical entities, by patiently explaining to me, time after time after time, that three frogs plus two grannies is not equal to five.

Mechanics of Mathematics

> *Only … Mathematics remains certain and verified to the limits of certitude and verification. Therefore all other sciences must be known and certified through mathematics* — *Roger Bacon*

Material presented in this book is aimed for the individuals with a very limited knowledge of mathematics or those who had a formal education but many years ago and need to be reminded about the mechanics of doing fundamental mathematics.

The main objective of the book is to introduce or reintroduce the fundamental concepts of mathematics to the practising engineers, managers and analysts who wish to make their day-to-day decisions based on the quantitative scientific principles.

The material covers the following mathematical topics:

- Arithmetic
- Algebra
- Functions
- Calculus
  - Differentiation
  - Integration
- Matrices
- Sets

In summary, this book could be beneficial in refreshing your mathematical skills or providing you with a new once, in order to study science and engineering in general and Mirce Mechanics in particular.

Enjoy the journey, as like everything else in life; the more you put in it the more pleasure it gives you back. Yes, it is applicable to maths too.

I wish to dedicate this book to the memory of my nana Katarina, who from the very early days of my life taught me, among million other things, the difference between adding numbers and adding physical entities, by patiently explaining to me, time after time after time, that three frogs plus two grannies is not equal to five.

Dr J, Knezevic,
Woodbury Park,
26th March 2013

Content

Chapter 1: Mathematics as a Language of Science

> *"The Universe is the grand book of science. The book lies continually open to man's gaze, yet none can hope to comprehend it who has not first mastered the language in which it has been written. This language is mathematics "*
>
> Galileo Galilei 1564-1642.

Mathematics is the science of structure, order, and relation that has evolved from elemental practices of counting, measuring, and describing the shapes of objects. It deals with logical reasoning and quantitative calculation, and its development has involved an increasing degree of idealization and abstraction of its subject matter. Since the 17th century, mathematics has been an indispensable adjunct to the physical sciences and technology, and in more recent times it has assumed a similar role in the quantitative aspects of the life sciences.

Chapter 2: Arithmetic

"A branch of mathematics that deals with numbers and computing with numbers under four operations of addition, subtraction, multiplication and division.

Webster's Comprehensive Dictionary

Arithmetic is a branch of mathematics in which numbers, relations among numbers and operations on numbers (that is, the processes of addition, subtraction, multiplication, division, rising to powers and extraction of roots) are studied and used to solve problems. [Greek, *arithmetike*, means counting]

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Table1. Examples of arithmetic rules

Operation	*8 x 5 – 3*	*20 – 9 + 4*	*6 – 9*	*-5 x(–3)*	*15 –12/3*	*6/(-2)*	*18/3 – 1*
Result	*37*	*15*	*-3*	*15*	*11*	*-3*	*5*

Thus, the arithmetic rules give us order of this is the order of doing things. With 8x5–3, the multiplication 'x' must be done first, which makes 40–3, which makes 37.

Multiplying and dividing must be done before adding and subtracting when there is nothing to tell us otherwise (like brackets, for example), as illustrated by the following two examples:

15 – 12/3 = 15 - 4 = 11 and 18/3 – 1 = 6 – 1 = 5.

In the cases were there are more than one multiplication and division or more than one addition and subtraction the conventional way is to work from the left.

For example: *15–9+4=6+4=10 and (35/5)*x*2=7*x*2=14*

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2.7 Brackets

Operation	$(3+4)^2$	$-(2\times3-11$	$(-3)^2$	$6(21-16)$	$6\times(10-3\times3)$	$\frac{25-10}{5-2}$	$\frac{4-11}{2}$

Result	*49*	*5*	*9*	*30*	*6*	*5*	*-7/2*

Brackets have to assemble the inner one before you can put it into the next. For example, in $(3+4)^2$ the correct calculation is

In the next example, *-(2x3–11)=-(6-11)=–(-5)=5*

In the third example it is important to notice that as "the brackets hold things together", $(-3)^2$ means that both the minus and the 3 are multiplied by themselves, giving *(-3* x*(-3)=+9.*

Brackets come in different forms, like $(\ ),[\ ],\{\ \},\ldots$. However, irrespective of the form they all have the same meaning. However, the same types of brackets written in different form have a different meaning. For example $\{[(\ )]\}$, means that the calculation has to be performed in the following order: small bracket, square bracket and finally curly bracket.

For example: $(2)[2]\{2\}=8$, whereas $\left\{\left[(2)^2\right]^2\right\}=\left\{[4]^2\right\}=16$

The line over the top is acting as the bracket. Another place where a line is used this way is in fractions:

$$\frac{25-10}{5-2}=(25-10)\Big/(5-2)=15\big/3=5$$

The actual shape of the brackets is not important. However, lleaving brackets out can be just as important as putting them in: $(-3)^2=(-3)\times(-3)=9$ whereas in $-3^2=-(3\times 3)=-9$

In summary, multiplying and dividing have to be done before adding and subtracting and calculations in inner bracket precedes calculations in outer brackets.

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Examples 2.2:

2^2 x $2^3=2^{2+3}=2^5$

2. *4^3 x $4^4=4^{3+4}=4^7$*
3. *7^3 x $7^{-2}=7^{3-2}=7^1=7$*

- Rule 3: $a^n \div a^m = \dfrac{a^n}{a^m} = a^n \times a^{-m} = a^{n-m}$

Examples 2.3:

1. $\dfrac{2^4}{2^2} = 2^{4\text{-}2} = 2^2$ *check* $\dfrac{2^4}{2^2} = \dfrac{2\times2\times2\times2\times}{2\times2} = 2\times2 = 2^2$

2. $\dfrac{(\text{-}4)^5}{(-4)^3} = (\text{-}4)^{5\text{-}3} = (\text{-}4)^2$ *check* $\dfrac{(\text{-}4)\times(\text{-}4)\times(\text{-}4)\times(\text{-}4)\times(\text{-}4)}{(\text{-}4)\times(\text{-}4)\times(\text{-}4)} = (\text{-}4)\times(\text{-}4) = (\text{-}4)^2$

3. $\dfrac{(1/3)^3}{(1/3)^2} = (1/3)^{3\text{-}2} = (1/3)^1 = (1/3)$ *check* $\dfrac{(1/3)^3}{(1/3)^2} = \dfrac{(1/3)\times(1/3)\times(1/3)}{(1/3)\times(1/3)} = (1/3)$

Rule 4: $(a^n)^m = \underbrace{a^n \times a^n \times \cdots \times a^n}_{m\ \ times} = a^{nm}$

Chapter 3: Algebra

"The branch of mathematics that treats of quantity and number in abstract, and in which calculations are performed by means of letters and symbols.

[Arabic aljebr the reunion of broken parts]

The history of algebra began in ancient Egypt and Babylon, where people learned to solve linear ($ax = b$) and quadratic ($ax^2 + bx = c$) equations, as well as indeterminate equations such as $x^2 + y^2 = z^2$, whereby several unknowns are involved. The ancient Babylonians solved arbitrary quadratic equations by essentially the same procedures taught today. They also could solve some indeterminate equations.

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Example 3.1:

Solve the following equation: $x^2 - 5x = -6$

Solution 3.1:

This is an equation with one unknown, namely x. Thus, our job is to find the possible values of x using some algebraic manipulations. Here is one way of doing it. Adding 6 on both sides of the equation, the following expression is obtained:

$x^2 - 5x + 6 = -6 + 6$

$x^2 - 5x + 6 = 0$

Now, we can write *5x* as *2x + 3x* in order to factor the equation. Thus,

$x^2 - 2x - 3x + 6 = 0$

$x^2 - 2x - 3x + 3x2 = 0$

$x(x - 2) - 3(x - 2) = 0$

Again, as we can see, $(x - 2)$ is common in the two terms of above equation. And hence, taking (x – 2) as a factor, we can write it as $(x - 2)(x - 3) = 0 \Rightarrow (x - 2) = 0$ and $(x - 3) = 0 \Rightarrow x = 2$ and $x = 3$

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3.3.1 Simplification

As one must have noticed in example 3.2.2 and 3.2.4, if a common denominator exists in both numerator and denominator of a fraction, it can be cancelled out using the inverse axiom for multiplication.

(1) $\frac{8}{2} = \frac{\cancel{2} \times 4}{\cancel{2} \times 1} = \frac{4}{1} = 4$

(2) $\frac{4ab}{2a} = \frac{\cancel{2}\cancel{a} \times 2b}{\cancel{2}\cancel{a}} = 2b$

(3) $\frac{-3xy^2}{xy} = -\frac{x\cancel{y} \times 3y}{xx\cancel{y}} = -3y$

(4) $\frac{-21}{-9} = +\frac{\cancel{3} \times 7}{\cancel{3}.3} = \frac{7}{3}$

(5) $\frac{x^2 + 3xy}{xy + 2xy^2} = \frac{x(x + 3y)}{xy(1 + 2y)} = \frac{x + 3y}{y(1 + 2y)}$

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Examples 3.5:

(1) $\left(\frac{4}{3}\right)\left(\frac{2}{5}\right) = \frac{(4 \times 2)}{(3 \times 5)} = \frac{8}{15}$

(2) $\left(\frac{-7}{2}\right)\left(\frac{9}{5}\right) = \frac{(-7 \times 9)}{(2 \times 5)} = -\frac{63}{10}$

(3) $\left(\frac{11}{6}\right)\left(\frac{2}{-3}\right) = \frac{(11 \times 2)}{6 \times (-3)} = -\frac{22}{18} = -\frac{11}{9}$

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Examples 3.7:

(1) $\dfrac{x^2+3x}{xy} = \dfrac{x(x+3)}{xy} = \dfrac{x+3}{y}$ (2) $\dfrac{ab-2a^2}{3b^2-6ba} = \dfrac{a(b-2a)}{3b(b-2a)} = \dfrac{a}{3b}$

(3) $\dfrac{4y+16y^2-8y^3}{2y-4y^2} = \dfrac{4y(1+4y-2y^2)}{2y(1-2y)} = \dfrac{\cancel{2y}\times 2(1+4y-2y^2)}{\cancel{2y}(1-2y)} = \dfrac{2(1+4y-2y^2)}{(1-2y)}$

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Example 3.10:

Expand the expression (2 + x)^5

Solution 3.10:

Setting $a = 2$ and $b = x$ in the general binomial expansion, the solution is:

$$
\begin{aligned}
(2+z)^5 &= \sum_{r=0}^{5}\binom{5}{r}2^{5-r}z^r \\
&= \binom{5}{0}2^5 + \binom{5}{1}2^4 z + \binom{5}{2}2^3 z^2 + \binom{5}{3}2^2 z^3 + \binom{5}{4}2z^4 + \binom{5}{5}z^5 \\
&= \frac{5!}{0!(5-0)!}(2^5) + \frac{5!}{1!(5-1)!}(2^4)z \\
&\quad + \frac{5!}{2!(5-2)!}(2^3)z^2 + \frac{5!}{3!(5-3)!}(2^2)z^3 + \frac{5!}{4!(5-4)!}(2)z^4 + \frac{5!}{5!(5-5)!}z^5 \\
&= 1\times 2^5 + 5\times 2^4 z + 10\times 2^3 z^2 + 10\times 2^2 z^3 + 5\times 2z^4 + 1\times z^5 \\
&= 32 + 80z + 80z^2 + 40z^3 + 10z^4 + z^5
\end{aligned}
$$

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Chapter 4: Functions

"The flower of modern mathematical though – the notion of a function"

Thomas J. McCornick,

The concept of function is the basic mathematical object that scientists and mathematicians use to describe relationships between variable quantities. Many scientific and engineering principles describe how one quantity depends on another.

The emergence of a notion of function as an individualized mathematical entity can be traced to the beginnings of infinitesimal calculus. Descartes (1596-1650) clearly stated that an equation in two variables, geometrically represented by a curve, indicates dependence between variable quantities. The idea of derivative came about as a way of finding the tangent to any point of this curve.

Newton (1642-1727) was one of the first mathematicians to show how functions could be developed in infinite power series, thus allowing for the intervention of infinite processes. He used "fluent" to designate independent variables, "relata quantitas" to indicate dependent variables, and "genita" to refer to quantities obtained from others using the four fundamental arithmetical operations. However, Leibniz (1646-1716) was first used the term "function" in 1673. He took function to designate, in very general terms, the dependence of geometrical quantities such as subtangents and subnormals on the shape of a curve. He also introduced the terms "constant," "variable," and "parameter." With the development of the study of curves by algebraic methods, a term to represent quantities that were dependent on one variable by means of an analytical expression was increasingly necessary. Finally, "function" was adopted for that purpose in the correspondence interchanged by Leibniz and Jean Bernoulli (1667-1748) between 1694 and 1698. The idea of a function was further formalized by Leonhard Euler (1707-1783) who introduced the notation for a function, y = f(x).

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Example 4.1:

For the functions $f(x) = x - 3$ and $g(x) = 1 + \sqrt{x-2}$ find the formulas and domains for the functions *f+g, f-g, fg* and *f/g*

Solution 4.1:

$$(f+g)(x) = f(x) + g(x) = (x-3) + \left(1+\sqrt{x-2}\right) = x - 2 + \sqrt{x-2}$$
$$(f-g)(x) = f(x) - g(x) = (x-3) - \left(1+\sqrt{x-2}\right) = x - 4 - \sqrt{x-2}$$
$$(fg)(x) = f(x)g(x) = (x-3)\times\left(1+\sqrt{x-2}\right)$$
$$(f/g)(x) = f(x)/g(x) = (x-3)/\left(1+\sqrt{x-2}\right)$$

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For example,

$log_{10}1000=3$, since $10^3=1000$

$log_5 25 = 2$, since $5^2 = 25$

$log_3 9 = 2$, since $3^2 = 9$

$log_3(1/27) = -3$, since $3^{-3} = 1/27$

$log_{1/4}16 = -2$, since $(1/4)^{-2} = 16$

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Example 4.2:

$log_2 8=3$ since *$2^3=8 \Rightarrow log_2(2\times4) = 3$* since *$2\times4 = 8$.*

Also, *$log_2 2=1$, $log_2 4=2 \Rightarrow log_2 2+log_2 4=3$*. Thus, *$log_2(2\times4)=log_2 2+log_2 4$.*

$$\log(1000) = \log(10\times100) = \log(10) + \log(100) = 1 + \log(10\times10) = 1 + \log(10) + \log(10) = 1+1+1 = 3$$

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Examples:

$$\Gamma(2) = 1! = 1$$
$$\Gamma(5) = 4! = 4\times3\times2\times1 = 24$$
$$\frac{\Gamma(6)}{2\Gamma(3)} = \frac{5!}{2\times2!} = \frac{5\times4\times3\times2\times1}{2\times(2\times1)} = \frac{120}{4} = 30$$

$$\frac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} = \frac{\frac{3}{2}\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} = \frac{\frac{3}{2}\times\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} = \frac{3}{4}$$

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Chapter 5: Calculus

"A method of calculating by use of highly specialised system of algebraic symbols."

[Calculus - Latin, pebble, used in counting]

It is a branch of mathematics that enables the manipulation of and working with quantities that vary continuously. It involves two fundamental operations: differentiation and integration.

Historically, integration was discovered first with ideas and results dated as back as ancient Greece when the method of exhaustion was developed to calculate the area of a region bounded on one side by a curve. Archimedes (287-212BC) used it to obtain the exact formula for the area of a circle.

Differentiation was discovered very much latter, during 17th century when French lawyer and amateur mathematician Pierre de Fermat (1601-1665) used it to find the maximum and minimum points of some special functions. He noticed that the tangent must be horizontal at some points, and developed a method for finding them by slightly changing the variable in a single algebraic equation and then letting the change "disappear".

The connection between the two processes of determining the area under the curve and obtaining a tangent at a point on a curve was first realised in 1663 by Barrow, who was Newton's professor at Cambridge University. However, the methods and the techniques of calculus has been developed independently, at the same time, by the English physicist, astronomer and mathematician Isaac Newton (1642-1727) and the German philosopher, logician, mathematician and scientist Gottfried Wilhelm Leibniz (1646-1716). They develop the calculus as a way of dealing with change and motion in terms of the effects on a function of an infinitesimal change in the value of the independent variable. Today is understood in terms of limits of real functions. The *differential calculus* concerns the rate of change of a dependent variable and so the slope of the curve, whereas the *integral calculus* extends the notion of the sum of a finite number of discrete values of a function to a continuous function and thus allows the calculation of the area under the curve.

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Example 5.1:

Find the derivative of $f(x) = x^2 + 2x$ using the limit process.

Solution 5.1:

We know from the above description that the derivative for $y = f(x)$ using the limit process can be obtained using the following formula.

$$\frac{d}{dx}[f(x)] = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

In this case $y = f(x)=x^2+2x$. So in order to apply the above formula, it is necessary to find $\Delta y=f(x+\Delta x)$, which is:

$$\Delta y = f(x+\Delta x) = (x+\Delta x)^2 + 2(x+\Delta x) = x^2 + 2x\Delta x + (\Delta x)^2 + 2x + 2\Delta x \quad \text{Now,}$$

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{[x^2 + 2x\Delta x + (\Delta x)^2 + 2x + 2\Delta x] - [x^2 + 2x]}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2 + 2\Delta x}{\Delta x} = \frac{\Delta x(2x + \Delta x + 2)}{\Delta x} = 2x + \Delta x + 2$$

Now, taking the limit of the difference quotient *($\Delta y/\Delta x$)* as Δx approaches to zero, it is obtained:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} [\frac{\Delta y}{\Delta x}] = \lim_{\Delta x \to 0} [2x + \Delta x + 2] = 2x + 2 .$$

Thus, the derivative of the function f(x) is 2x + 2.

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Example 5.2:

Find the derivative of $f(x) = 4x^3$

Solution 5.2:

Taking k=4, $y=x^3$ and applying Rule 1we get, $f'(x)=4(3x^2)=12x^2$. Since $\frac{dy}{dx} = 3x^2$

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Example 5.5:

Find the derivative of $\frac{e^x}{x^2 + 2}$

Solution 5.5:

Taking $u = e^x$ and $v = x^2 + 2$ gives $\frac{du}{dx} = e^2$ and $\frac{dv}{dx} = 2x$

So by quotient rule: $\frac{d}{dx}(\frac{e^x}{x^2 + 2}) = \frac{(x^2 + 2)e^x - 2xe^x}{(x^2 + 2)^2}$

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Example 5.7:

Let $F(x)=2x$, show that $\int_2^4 f(x)dx = F(4) - F(2)$, where $f(x)$ is the derivative of $F(x)$.

Solution 5.7:

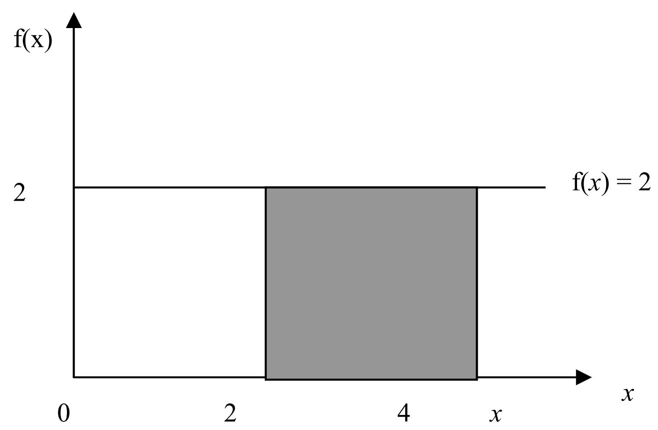

Fig 5.5 Area under the function f(x) = 2

$$f(x) = \frac{dF}{dx} = \frac{d}{dx}(2x) = 2 \qquad \int_2^4 f(x)dx = \int_2^4 2dx = 2x \Big|_2^4 = 2\times 4 - 2\times 2 = 4$$

The above procedure of integration will be clear once we know the rules of integration. Now to evaluate the right-hand side, we take the total function F(x) and substitute the values of the interval *[2, 4]* to find the difference. Thus,

$F(4) - F(2) = 2\times 4 - 2\times 2 = 8 - 4 = 4$

$$\int_2^4 f(x)dx = F(4) - F(2) = 4$$

Thus, we just proved that the area under the function *f(x)=2* between the points *2* and *4* is equal to the difference of the total function *F(x)=2x*, evaluated at those two points.

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For example,

$$\int_1^2 (3x+4)dx = \frac{3}{2}x^2 + 4x + c \Big|_1^2 = \left[\frac{3}{2}(2)^2 + 4(2) + c\right] - \left[\frac{3}{2}(1)^2 + 4(1) + c\right]$$

$$= 6 + 4 + c - \frac{3}{2} - 4 - c = 6 - \frac{3}{2} = \frac{12-3}{2} = \frac{9}{2}$$

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Example 11:

Find the convolution of f and g if *f(t) = tu(t*) and *g(t) = t2u(t).*

Solution 11:

$f(t-x) = (t-x)u(t-x)$ and $g(x) = x^2u(x)$. Therefore:

$$(f*g)(t) = \int_0^t (t-x)x^2dx = \left[\frac{1}{3}x^3t - \frac{1}{4}x^4\right]_0^t = \frac{1}{3}t^4 - \frac{1}{4}t^4 = \frac{4}{12}t^4 - \frac{3}{12}t^4 = \frac{1}{12}t^4$$

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Chapter 6: Matrix

> *"A rectangular array of symbols or terms enclosed between parentheses or double vertical bars to facilitate the study of relationships.*
>
> [Latin, matris, mother]

The solution of simultaneous equations is part of elementary algebra. Many science and engineering problems can be formulated in terms of simultaneous equations, but in most practical situations the number of equations is extremely large and traditional method of solution is not feasible. Even the question of whether solutions exist is not easy to answer. Setting the equations up in matrix form provides a systematic way of answering this question and also suggests practical methods of solution. For example, we know that the equation of a plane can be written in the form

$$\alpha x + \beta y + \gamma z = p$$

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Example 6.4:

Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Find where possible,

1) $A + B$ *2) $A + C$* *3) $C - A$* *4) $3A$*

5) $4B$ *6) $C + B$* *(7) $A^T + A$* *8) $A + C^T + B^T$*

Solution 6.4:

1) $A + B$ is not possible, as they are not of the same order, A is 3x3 and B 3x2

2) $$A + C = \begin{bmatrix} 1+0 & 2+1 & 1+1 \\ 1+0 & 1+0 & 2+1 \\ 1+1 & 1+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{bmatrix}$$

3) $$C - A = \begin{bmatrix} 0-1 & 1-2 & 1-1 \\ 0-1 & 0-1 & 1-2 \\ 1-1 & 0-1 & 0-1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$$

4) $$3A = \begin{bmatrix} 3 & 6 & 3 \\ 3 & 3 & 6 \\ 3 & 3 & 3 \end{bmatrix}$$

5) $4B = \begin{bmatrix} 8 & 4 \\ 4 & 0 \\ 4 & 4 \end{bmatrix}$

6) C + B is not possible, as they are not of the same order, C is *3*x*3* and *B 3*x*2*

7) $A^T + A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 3 \\ 2 & 3 & 2 \end{bmatrix}$

8) A + C^T + B^T is not possible.

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Example 6.9:

Find A^{-1} and B^{-1} for the matrices

$(a) \quad A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \qquad (b) \quad B = \begin{bmatrix} 5 & 2 & 4 \\ 3 & -1 & 2 \\ 1 & 4 & -3 \end{bmatrix}$

Solution 6.9:

adj A = $\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$ and |A| = -1 so that $A^{-1} = \dfrac{adjA}{|A|} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$

$(b) \quad adjB = \begin{bmatrix} -5 & 11 & 13 \\ 22 & -19 & -18 \\ 8 & 2 & -11 \end{bmatrix}^T = \begin{bmatrix} -5 & 22 & 8 \\ 11 & -19 & 2 \\ 13 & -18 & -11 \end{bmatrix}$ and |B| = 49 so that

$$B^{-1} = \frac{1}{49}\begin{bmatrix} -5 & 22 & 8 \\ 11 & -19 & 2 \\ 13 & -18 & -11 \end{bmatrix}$$

In both cases it can be checked that AA^{-1} = I and BB^{-1} = I.

Note

Finding the inverse of a 2x2 matrix is very easy, since for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (Provided that ad - bc ≠ 0)

2. To find the inverse of a product of two matrices, the order is reversed:

$(AB)^{-1} = B^{-1} A^{-1}$ (Provided that A and B are invertible).

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Appendix 1:   The Concept of the set

Set theory is a branch of mathematics that deals with the properties of well-defined collections of objects, which may be of a mathematical nature, such as numbers or functions, or not. The theory is less valuable in direct application to ordinary experience than as a basis for precise and adaptable terminology for the definition of complex and sophisticated mathematical concepts.

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$A - B$. Set operations are summarised in Table A1.2.

Universal	Subsets	Operation	Symbol	New set
Z	A, B	Union	$\cup$	$A \cup B$
	A, B	Intersection	$\cap$	$A \cap B$
	A	Complement	‘	$A' = Z - A$
	A, B	Difference	-	$A - B$

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Closing Remarks

I do hope that you have enjoyed going through the book and that by doing it you found things that you still know, things that you knew but forgotten and some new things too. However, I also hope that you will be going to back to this book on many occasions in the future and will consider it as your good friend on whom you can always rely.

Also I do hope that after reading this text you will fully understand the following words of the great mathematician Georg Cantor (1845-1918) who said *“The essence of mathematics lies in its freedom.”* The dimensions of mathematical freedom are endless; from the topics of development to the citizenship as it does not have nationality or geographical coordinates.

Please feel free to pass any comments to me, regarding the possible improvements of the book, as all that I have done was to “package” the existing knowledge of mathematics, generated by thousand of generations over thousands of years into understandable collections of mathematical truths illustrated through numerous examples, in order for the reader to see the "mechanics" of their workings.

At the very end, I wish to emphasize that whatever is covered by this book is mathematically correct. However, there is much more that has not been addressed. It is necessary to stress that topics not covered could not be understood without understanding the topics covered in this book. Please remember the Chinese proverb that say's: *"The journey of million miles starts with the first step"*.

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